1 Estimation of Electricity Demand Elasticity Using a Bayesian Heteroskedastic SUR Model

1.1 Abstract

The paper I propose wants to investigate the consumers' reactivity to change in price in the Italian Wholesale Electricity Market which is based on an implicit auction model. The research question is "Are rationale consumers able to react to expected changes in price of electricity and what is the extent of buyers' elasticity?"

The data I used refer to hourly bids occurring in the Day A-head Market, bids were aggregated in a price descending order to form the Marshallian demand for each hour of the day. Records pertain the 2011.

Economic model lies in the neoclassical framework. The duality approach is exploited to legitimately shift from the Hicksian demand (derived from the first conditions of optimization problem) to the Marshallian demand in which quantity are function of price and total expenditure .

The log-linear function is used as functional form of the hourly aggregated demand where its logarithm is a function of the logarithm of the corresponding prices, adjusted by the monthly consumer index price and dummy variables approximating the total expenditure**.** The parameters related to the logprice vectors represent elasticities and hourly elasticities for each day of the year are obtained by adjusting the parameters through the estimates of daily iteration dummy's coefficients.

I want to analyse if economic agents, given rational expectation of changes in prices are able to modify their electricity profiles. For this reason I divided each day into two groups of hours,one ranging from 9 a.m. to 8 p.m. (peak hours) and the other from 9 p.m. to 8 a.m (off-peak hours), then, I blocked together the hourly aggregated demand vectors referring to the same group of hours. The assumption of correlation depends on the fact that consumers and traders in the day a-head market can set prices in advanced and, given rational expectation of changes in price, they are able to react to high prices and to adopt a strategic behaviors affecting market price sensitivity such as postponing their electricity consumption, rescheduling their activities and their demand profiles, flattening the load curves within the specific period of the day.

Given this market structure Bayesian Heteroskedastic Seemingly Unrelated Regression model is applied:

$$\underset{(nm\times 1)}{Y} = \underset{(nm\times K)}{X} \cdot \underset{(K\times 1)}{\beta} + \underset{(nm\times 1)}{\varepsilon}$$

$$\varepsilon = [\varepsilon_1 ... \varepsilon_m]';\ E(\varepsilon) = 0;\ Var(\varepsilon) = \begin{bmatrix} h^{-1}\Lambda_1^{-1} & 0 & ... & 0 \\ 0 & h^{-1}\Lambda_2^{-1} & 0 & \\ & & ... & \\ 0 & & 0 & h^{-1}\Lambda_m^{-1} \end{bmatrix}$$

$$\varepsilon_j|\Lambda_j \sim N(0, h \times \Lambda_j) \qquad \text{for } j = 1, ...m$$

where Y is the vector of the logarithm of the aggregated demands, X the matrix of the explanotory variables (containing the logarithm prices vectors and the dummy variables), n are the number of observations i in each hours and $m = 1, ..., 12$ the number of equations (equal to the number of hours in the peak and off-peak blocks).

Given unknown heteroskedasticity assumption the model is hierarchical since the unconditional distribution of the error terms $p(\varepsilon_i)$ is a Student-t distribution (with ν_λ degree of freedom).

In the Bayesian framework we want to assess which random mechanism generates the data, given the sample. The parameters indexing the distribution generating data are in fact random variables. This randomness depends on the fact we have uncertain knowledge about their values and this uncertainty is expressed giving a probability distributions to them. These distributions are called priors and they are derived from pre-experimental information. After the sample is drawn, the likelihood function (the conditional probability of the sample given the parameter) is computed. Then, these two main features of bayesian inference, prior and likelihood, are combined together using Bayes Theorem leading to the posterior distributions which describes how prior knowledge are updated by the sampling. They are the distribution of the parameters after observing the sample and they repesent the assessment of where the true values of parameters are likely to lie in the parameter space.

The parameters to be estimated are the vector $[\beta_1, ..., \beta_m]'$, the precision h, the elements of error terms' covariance matrix Λ_j (with iand the degree of freedom of the student-t ν_λ. The prior distributions are a Multivariate normal for β, a gamma density for h and the degree of freedom of the Student-t ν_λ and a Wishart distribution for Λ_j.

After applying the Bayes Theorem the joint posterior distribution is derived; since it has not a well-known functional form, it required to be simulated through MCMC method. The simulation procedure combined Metropolis Hasting algorithm with the Gibbs Sampling and led to the approximation of the parameters' joint posterior distribution : $p(\beta, \nu_\lambda, h|y)$; moreover, the mode of the marginal posterior distribution $p(\beta|h, \nu_\lambda, y)$ can be computed in order to derive some summary statistics about price elasticity.

The inferential procedure leads to two main results; first, there exist a well defined and statistically robust value for the demand elasticity, which can be estimated on average (in absolute value) 0.04 to 0.14; second, elasticity values depend crucially on time of day and some characteristics of the market structure. Notably, estimated elasticities are generally higher (in absolute value) during peak hours, when there is no market congestion, and when there are high market equilibrium price levels.